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LETTER TO THE EDITOR

COMMENTS ON "AN EXPERIMENTAL STUDY OF THE STABILITY OF LIQUID-FLUIDIZED BEDS"

The experimental results in the above paper (Ham *et al.* 1990) appear to have a much simpler explanation than the one offered by the authors.

Table 1 in the above paper shows that the void fraction at minimum fluidization varied between 0.398 and 0.447 with a mean value of 0.415. Since ϵ_{mf} is essentially geometrical, describing the random packing of particles, this range of values is presumably due to the lack of complete geometric similarity, the biggest (small) variation being probably ascribable to out-of-roundness of the plastic particles.

Table 4 in the paper shows that significant fluctuations emerged above ϵ_c in the range 0.41–0.464, with an average value of 0.436. The scatter is less than for ϵ_{mf}. The simple conclusion is that the bed expanded by a few percent before becoming sufficiently mobile to support disturbances. Indeed, it would be fair to adopt the criterion $\epsilon_c \cong 0.44$ with no more precision being justifiable in light of the uncertainty in geometric similarity previously mentioned.

The quantity Q in [A.7] is a modified Froude number interpreted by the authors as a dimensionless elasticity that is inferred from a stability analysis. The more straightforward explanation is that the results shown in figures 7–10 represent a correlation between Froude number and Reynolds number (Re), at a void fraction $\epsilon \cong 0.44$, which describes the equilibrium between buoyancy and drag and has nothing to do with dynamic behavior at all. This may be checked by using the authors' data to compute Q and Re at $\epsilon = 0.44$, using simple interpolation. The results are shown in figure 1, together with the line from the authors' figure 7. Clearly the data in figure 7 barely move when expressed this way. The correlation is every bit as good as in the original figure. Indeed, almost as good agreement is obtained in figure 1 by plotting Q and Re at incipient fluidization, a condition that again has no direct connection with either stability or elasticity.

How does this come about? If the criterion [A.7] is used, the correlation on p. 182 becomes

$$\frac{u_c^2 \rho_f}{gd(\rho_s - \rho_f)} f(\epsilon_c, n, R) = \mathrm{Re}_c^{0.77} R^{0.23}, \qquad [1]$$

where "f" is a weak function of R and Re (which influences n) but contains no influence of ϵ_c if this is constant. (Presumably the authors' ρ_p and ρ_s are identical.)

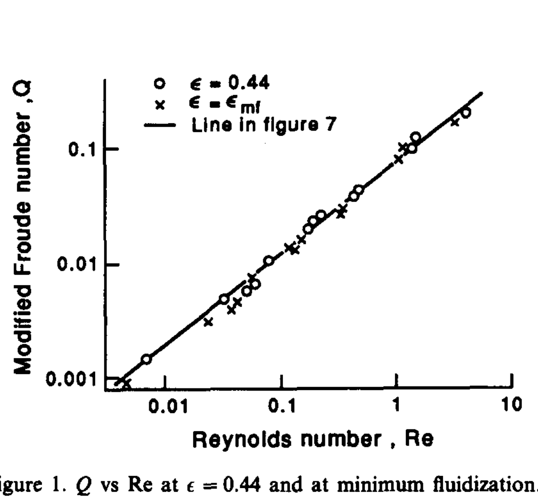

Figure 1. Q vs Re at $\epsilon = 0.44$ and at minimum fluidization.

Now, the equilibrium between drag and buoyancy in a uniformly fluidized bed can be represented (Wallis 1969) by

$$\frac{u^2\rho_f}{gd(\rho_s-\rho_f)}g(\epsilon)=\frac{1}{C_d}, \qquad [2]$$

where $g(\epsilon)$ is constant if ϵ is constant and C_d is a drag coefficient that depends on Re by way of an equation such as [A.14]. At very low Re, $C_d \sim 1/\mathrm{Re}$ and [2] becomes at $\epsilon_c \cong 0.44$ (or any other constant value such as ϵ_{mf}):

$$\frac{u^2\rho_f}{gd(\rho_s-\rho_f)} \sim \mathrm{Re}, \qquad [3]$$

which explains figure 8(b). Over a wider range of Re, [A.14] or some similiar correlation may be approximated by $C_d \sim \mathrm{Re}^{-m}$, where $m < 1$, as in the authors' value 0.77, rendering [1] and [2] equivalent. There is also some influence of n's dependence on a weak power of Re. Figure 7 is therefore indeed a correlation of the fluid flux needed to produce $\epsilon \cong 0.44$.

The Re modified by R achieves better success in figure 10 because this compensates approximately for the factor ρ_s rather than ρ_f in the numerator of Q when [2] is compared with [A.7]. However, it would be more reasonable to leave Re alone and modify Q to make it more closely resemble the Froude number on the l.h.s. of [2]. Since the factor in square brackets in [A.7] does not vary much, the required factor is approximately R. Rather than using R itself, which would move both the line and the points in figure 9, it is simpler to leave the line in place and multiply the authors' Q by the density ratio of the particles. This has the property of raising the open points in figure 9 by a factor of $2.47/1.19 = 2.08$ and lowering the solid point by a factor of $2.47/4.14 = 0.6$, which does indeed improve the correlation.

It is doubtful if any significant further conclusion can be reached from these data, either about Batchelor's theory or about the compressibility of a fluidized bed.

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RESPONSE

In his letter, Professor Wallis suggests that our recent experiments on stable fluidization have a simpler interpretation than the one we ascribe to them, namely that the systems studied exhibit an instability whenever the bed expansion reaches $\epsilon \approx 0.44$. A plot of Q vs Re is given in support of this interpretation of the experiments. His interpretation is incorrect on several accounts:

1. The mode of this instability is well-known to be a one-dimensional wave of *dilation*. Simple physical arguments and direct observations (El Kaissy & Homsy 1976) show that the particle motion consists of vertical oscillations about a mean position. Contrary to Wallis' hypothesis, particles will *always* be "sufficiently mobile" to allow this mode of motion. Other modes involving lateral shearing motions of the particles may indeed be stabilized by a yield stress associated with dense packing. We are not unaware of this possibility, which one of us (G.M.H.)